Models of Centralized Inventory for Single-Vendor and Single-Buyer System with Controllable Leadtime and Batch of Production

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Abstract

This research will be developed a model of centralization between vendor-buyer with probabilistic demand ,lot quantities, leadtime, frequency of delivery and batch of production as one of the decision variables. The objectives of this research were total cost minimization on vendor and buyer. The mathematical model was developed by attempting various inputs on the model and comparing the results for each variation of the model inputs to obtain mutually beneficial policies on vendors and buyer. Lead time can be reduced by adding crashing costs (extra costs incurred to shorten lead time), so lead time can be controlled. Benefits from reduced lead times are low safety stocks, reduced stock outs, improved service levels to consumers, and provide competitive advantage, as evidenced by Just-In-Time (JIT) production. At the end, the authors evaluate the advantages of the coordination strategy offered by numerical examples.This paper describes one types of models with controlled lead times, a model with centralized decision model. The solution to be given is the optimal solution.

Keywords:centralized inventory, controllable leadtime, backorder,batch of production

1. Introduction

This paper describes the centralized model with controlled lead times and batch production. The solution to be given is the optimal solution. In most literature on the amount of economical ordering (EOQ), lead time is seen as a constant or stochastic variable that uses a deterministic or probabilistic model. But in a model that becomes very unrealistic is often lead time assumed as an uncontrolled variable. According to Tersine, lead time consists of components:

- Order preparation time
- Ordering time
- Leadtime from vendor
- Delivery time
- Set up time

Lead time can be reduced by adding crashing costs (extra costs incurred to shorten lead time), so lead time can be controlled. Benefits from reduced lead times are low safety stocks, reduced stock outs, improved service levels to consumers, and provide competitive advantage. As evidenced by Just-In-Time (JIT) production. Reduction of lead time is seen as an effective way to realize the rapid response of the entire supply chain and one of the most important sources in competitive advantage.

Research Position

Goyal (1976) is the first researcher to model buyer and supplier coordination called Joint Economic Lot Size model. The solutions generated from this model can provide significant savings on total combined inventory costs. Pujawan and Kingsman (2002) develop a supplier-buyer supply model for an unlimited time horizon. In this model the buyer wants the shipment to be done n times, while the production made by the supplier is m times. If the delivery is done in q quantity, then the buyer's ordering lot is formulated as nq and the production lot as mq. The researchers compared the lot stream model with no lot streaming for 2 different cases, ie: (i) if the decision was made by each party, and (ii) if the decision was made jointly. The solutions obtained show that good synchronization between supplier and buyer in determining delivery frequency and production time will result in significant savings on total inventory cost. Some of the above models also still assume a deterministic demand, whereas in real conditions the demand will vary over time. For that reason Jauhari (2009) makes inventory models in real conditions that try to relax the assumptions of deterministic demand into probabilistic demand, but do not consider leadtime and backorder. Therefore, in this research will be developed a model of coordination between buyer and supplier which with controllable leadtime and backorder policy. Expected from this model can minimize the total cost of buyers and vendors, and also balances the total cost between the two.

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System Analysis

In this study,the issues discussed are one vendor and one buyer. In the supply chain usually consist of suppliers, manufacturers, retailers, and consmers.There is a flow of physical, information, and finance, process starts with the raw materials produced by the supplier and ends with the finished product consumed by the consumer. The supply chain network can consist of one vendor model and one buyer in known conditions.

The objective of the research is to optimize the supply system in the supply chain by controlling the lead time of the decentralized mode. With the centralized mode (centralized mode) is the expected total inventory cost on buyers and vendors can be balanced. Which with the decentralized mode in the previous paper there is an imbalance of total cost inventory between buyer and vendor. Illustration can be seen in figure 1 below.

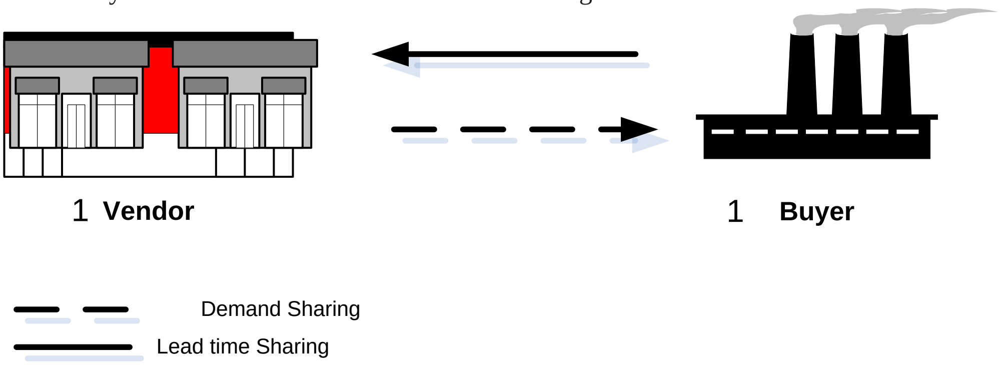

Fig. 1. Two Echelon Inventory Problem

Structural Aspects **:** 1 vendor – 1 buyer
Functional Aspects **:**Relationship between vendor and buyer
Evironment **:** Ignored
Objective **:**Minimize total cost of vendor and buyer

2. Method

Table 1. Component Model

Component Model	Fei Ye danYi Na Li (2008)	This research
Problem	Determining optimal inventory policy	How to determine the optimal inventory policy?
Criteria Performance	Expectation of total minimum cost (buyer and vendor)	Expectation of total inventory cost for vendor and total inventory cost for buyer
Decision Variable	Order lot size*(Q)*	Order lot size (q)
	Lead Time *(L)*	Lead time (L)
		Frequency of delivery (n)
		Batch of production (m)
Parameter	Ordering cost *(A)*	Ordering cost (A) atau set up cost *(S)*
	Holding cost of buyer (hr)	Holding cost of buyer *(hr)*
	Holding cost of vendor (hs)	Holding cost of vendor *(hs)*
	Crashing cost (ci)	Crashing cost (ci)
	Price/unit (P)	Shortage cost (γ)
	Shortage cost (γ)	Delivery cost (f)
		Price/unit (P)
Constraint	-	-

Model Formulation

Notation mathematical

D = Average demand/year
p =vendor production level (P > D)
A =Ordering cost for buyer
h_r = Holding cost for buyer
h_s = Holding cost for vendor
S = Set up cost
q = Optimal lot size (decision variable)
L = Lead time (decision variable)

γ =Shortage cost
f =Delivery cost
k = Safety factor
δ= Deviation standard

The following assumptions that used for this research :

a. Supply chain of two echelons consists of a single vendor and a single buyer
b. Production rate of the supplier is assumed to be P, where the production rate is greater than demand rate (P>D)
c. Inventory is continuously replenished where refilling is determined by its point reorder (r).
d. The demand during lead time L is assumed to be normal distribution with mean = uL and standard deviation = δ√L and k is safety factor, shortages inventory is fullfiled with back order
e. Lead time has n independent components. The*i*thcomponent has a minimum duration$a_i$and normal durationb_i, buyer's crashing costc_i, and vendor's crashing cost d_i. To simplify we can arrange $c_i$dand$d_i$like $c_1 \le c_2 \le ...c_n$. and $d_1 \le d_2 \le ..... d_n$. So it can bee seen clearly that to reduction of lead time, it should be first on component 1 (cause it has minimum crashing cost) and then component 2, and so on.
f. If $L_0 = \sum_{j=1}^{n} b_j$and $L_i$is the length of leadtme at component 1, 2,...,icrashed to the minimum duration then $L_i$is expressed as

$$L_i = \sum_{j=1}^{i} a_j + \sum_{j=1+1}^{i} b_j = \sum_{j=1}^{i} b_j - \sum_{j=1}^{i}(b_j - a_j) = L_0 - \sum_{j=1}^{i}(b_j - a_j) \quad \textbf{(1)}$$

i= 1,2,...n

g. buyer orders a number of products *nq* to vendor withthe delivery frequency of*n* time (based on buyer's need) with lot of delivery*q*, so to fullfilbuyer's demand, vendor produce the productwith batch production*mq*. Delivery of product from vendor to buyer is done every period(*nq/D*) and can be done if vendor has minimum inventory*q*, so no need to wait for all batches to be produced

Mathematical model

a. Buyer's inventory cost model

Based on assumption above, so year cost expectation for buyer :

TEC_r = Ordering cost + Holding cost + Leadtime crashing cost + Shortage cost

- **Ordering cost (Op)**
  In this model buyer orders the product a number of *nq* to vendor with delivery frequency of *n* time (base on buyer's need) and delivery lot*q*, so expectation of the ordering cost becomes :

$$Op = \frac{D}{nq}(A + fn) \quad \textbf{(2)}$$

- **Holding cost (O_{hr})**
  Because the model development is done by using backorder policy, so holding cost for buyer becomes:

$$Ohr = h_r\left(\frac{q}{2} + r - D(L_0 - \sum_{j=1}^{i}(b_j - a_j)\right) \quad \textbf{(3)}$$

  with*r* = reorder point

- **Leadtime crashing cost (Oc)**
  Crashing cost is a cost that must be issued by buyer because it can reduced leadtime. For that reason leadtime crashing cost of buyer can be searched with :

$$R(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i} c_j(b_j - a_j) \quad (4)$$

  Notation :
  c : buyer's crashing cost
  b : normal duration
  a : leadtime minimum duration

- **Shortage cost (Ok)**
  Ok = Shortage cost/unit X N

$$Ok = \gamma \frac{D}{q}\int_r^{\sim}(x - r)f(x)dx \quad \textbf{(5)}$$

$$\int_r^{\sim}(x - r)f(x)dx = SL(L_0 - \sum_{j=1}^{i}(b_j - a_j))[f(z_\alpha) - z_\alpha \varphi(z_\alpha)] \quad \textbf{(6)}$$

Total cost inventory for buyer becomes :

$$TEC_r = \frac{D}{nq}(A + fn) + h_r\left(\frac{q}{2} + r - D(L_0 - \sum_{j=1}^{i}(b_j - a_j)\right) + \frac{D}{q}R(L) + \gamma\frac{D}{q}\int_r^{\sim}(x - r)dx \quad (7)$$

b. Vendor's inventory cost model

Base on notation and assumption, so year cost expectation for vendor becomes :

TEC_s = Set up cost + holdin cost + leadtime crashing cost

- **Set up cost (Os)**

  Suppliers produce products with batch productionmq, so set up cost for vendor becomes:

$$Os = \frac{D}{mq}S \quad (8)$$

- **Holding cost (O_{hs})**

  To fullfil buyer's demand, vendor produc with batch production mq. The inventory level of vendor is obtained by reducing the accumulated production by the accumulation of buyer consumption. For more details can be seen in the figure below:

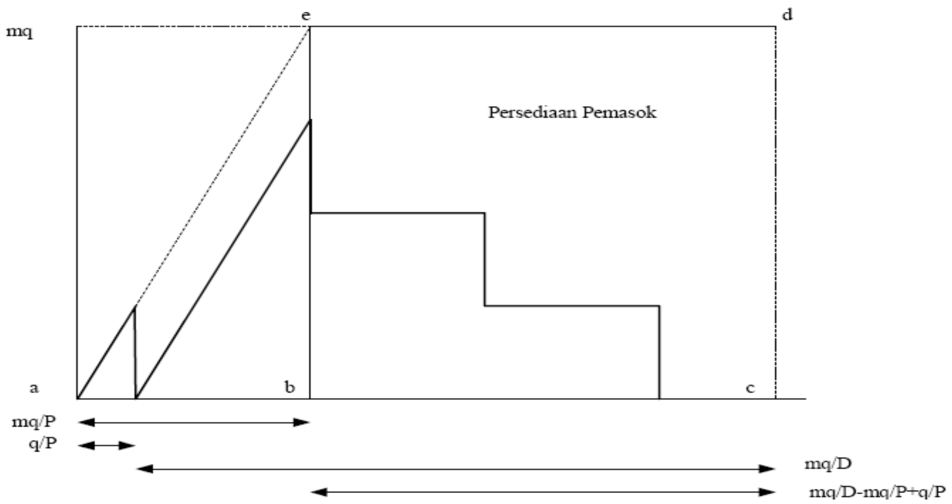

Fig.2. Holding cost

According to jauhari (2009) dan Ben-Daya (2004) holding cost for vendor becomes :

Total vendor's inventory = vendor production accumulation – buyer consumtion cumulation

Total vendor's inventory = $\left(\frac{mq}{2} - \frac{q}{2}\right) - \left(\frac{mq}{2} - q\right)\frac{D}{P}$

So holding cost for vendor becomes :

$$Ohs = h_s\frac{q}{2}\left[(m-1) - (m-2)\frac{D}{P}\right] \quad (9)$$

- **Leadtime crashing cost (Oc)**

  Crashing cost is a cost that must be issued by the vendor because it reduce leadtime.For that value of vendor's leadtime crashing cost can be searched with :

$$M(L) = d_i(L_{i-1} - L) + \sum_{j=1}^{i} d_j(b_j - a_j)\ , \quad (10)$$

Notation :

d : vendor's crashing cost/unit/time

b : normal duration

a : leadtime minimum duration

Expectation crashing cost/year for vendor :

$$Oc = \frac{D}{q} M(L) \tag{11}$$

notation :

m : the production undertaken by the vendor is as much as m times.

$$TEC_s = \frac{D}{mq} S + h_s \frac{q}{2}\left[(m-1) - (m-2)\frac{D}{P}\right] + \frac{D}{q} M(L) \tag{12}$$

c. Centralized Model

$$TEC_{Gab}(q, L) = TEC_r + TEC_s$$

$$TEC_{Gab} = \frac{D}{nq}(A + fn) + h_r\left(\frac{q}{2} + r - D\left(L_0 - \sum_{j=1}^{i}(b_j - a_j)\right)\right) + \frac{D}{q} R(L) + \gamma \frac{D}{q}\int_r^{\sim}(x - r)dx + \frac{D}{mq} S + h_s \frac{q}{2}\left[(m - 1) - (m-2)\frac{D}{P}\right] + \frac{D}{q} M(L) \tag{13}$$

Model Solution

a. Centralized Model

$$\frac{\partial TEC_{gab}(q, L)}{\partial Q} = 0$$

$$\frac{\partial TEC_r(Q,L)}{\partial Q} = -\frac{D}{nq^2}(A + fn) - \frac{D}{q^2} R(L) + \frac{1}{2} h_r - \frac{\gamma D}{q^2}\int_r^{\sim}(x - r)dx - \frac{D}{mq^2} S + \frac{1}{2} h_s \left[(m-1) - (m-2)\frac{D}{P}\right] - \frac{D}{q^2} M(L) = 0 \tag{14}$$

$$q^* = \sqrt{\frac{2D\left[\left(\frac{A}{n}+f\right)+R(L)+\gamma\int_x^r (x-r)f(x)\right]dx+\frac{S}{m}+M(L)}{h_r+h_s\left[(m-1)-(m-2)\frac{D}{P}\right]}} \tag{15}$$

$$\frac{\partial TEC_r(q,L)}{\partial r} = 0 \rightarrow\ h_r - \frac{\gamma D}{q}\int_r^{\sim}(x - r) f(x)dx = 0 \tag{16}$$

$$\int_r^{\sim} f(x)dx = \frac{h_r q}{\gamma D} \tag{17}$$

Thus the probability of shortage inventory can be expressed as:

$$\alpha = \frac{h_r q}{\gamma D} \tag{18}$$

Algorithm: set *m = 1*

Step 1 : Calculate initial $q_o$using formula:

$$q = \sqrt{\frac{2D\left[\left(\frac{A}{n} + f\right) + \frac{S}{m}\right]}{h_r + h_s\left[(m-1) - (m-2)\frac{D}{P}\right]}}$$

Step 2 :Based on the initial$q_o$above, we can find probability of shortage$\alpha$by using equation (17**)** and then we can calculate the values of*r1 ** and *N* (the number of shortage) using the following equation:

$$r1^* = D\left(L_0 - \sum_{j=1}^{i}(b_j - a_j)\right. + z_\alpha \delta\sqrt{L}$$

$$\int_r^{65} (x - r)f(x)dx = SL\left(L_0 - \sum_{j=1}^{i}(b_j - a_j)\right)[f(z_\alpha) - z_\alpha \varphi(z_\alpha)]$$

Step 3 : Recalculateq_{o1}using the equation:

$$q^* = \sqrt{\frac{2D\left[\left(\frac{A}{n}+f\right)+R(L)+\gamma\int_x^r (x-r)f(x)\right]dx+\frac{S}{m}+M(L)}{h_r+h_s\left[(m-1)-(m-2)\frac{D}{P}\right]}}$$

Step 4 : Repeat above steps until *q* and *r* values do not change greatly.

Step 5 : If $TC\ (q_o) \leq TC\ (q_{o2})$, then repeat steps 1 to 4 with $m = m + 1$ but if on the contrary then proceed to step 6.

Step 6 : Calculate the value of $TC\ (q_o) = TC\ (q_{o2})$, so we get the value of *q* and *m* optimal.

3. Results

D = 600 unit/year

P = 2500unit/year

h_r = \$20/ unit/year

A = \$200/ order

δ = 7 unit/ week

γ =\$60/ unit

h_s = \$40/ unit/year

S = \$250/ setup

f = \$25

Table2. Numeric Data

i	*bi* (day)	*ai* (day)	*ci* (day)	*di* (day)
1	20	6	0,4	0
2	20	6	1,2	2
3	16	9	5	3

Leadtime calculation (Leadtime crashing cost)

For $i = 0$

Lo = 8 week

R(L) = 0

For $i = 1$

$$L_1 = L_0 - \sum_{j=1}^{1}(b_1 - a_1)$$

L_1 = 8 – 14 day (2 week) = 6 week

Tabel 3. Total Cost (before model development)

i	L_i	R(L)	M(L)	Q_i	TEC_{sc}	y	
						TEC_s	TEC_r
0	8	0	0	136,57	4834,56	1753,85	3078,06
1	6	5,6	0	137,20	4747,07	1751,84	2995,23
2	4	22,4	28	143,44	4805,91	1851,357	2954,56

Table 4. Total Costafter model development (this research)

N	m	K	R(L)	M(L)	q	TEC_r	TEC_s	Total Cost
1	1	1,45	0	0	140	2451	1744	4195
2	1	1,5	5,6	0	125	1954	1800	3754
3	1	1,45	2,4	28	127	850	1925	275

4. Conclusion

From the calculation result can be seen that total cost of buyer and total cost of vendor to be reduced, and the difference in total cost between buyer and vendor to be reduced. The occurance of balance with reduced total cost of buyer and vendor by using this centralization method, so buyer and vendor no one is too loss. Inventory model that has been developed in this research can still be developed according to the characteristics of different problems. Existing models can also be developed into more complex issues such as multi buyer, multi vendor, multi product.

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